Epistemics of Quantum Mechanics – A Study of Ideas Held by Students and Teachers

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A diagnostic test has been administered to 47 postgraduate students, 12 undergraduate students, and 15 teachers, supplemented by interviews, to probe their conceptual difficulties and alternative thinking in interpreting, explaining and drawing inferences while answering qualitative questions in introductory quantum mechanics. We find that the respondents have common as well as individual difficulties in this domain. These identified difficulties have been analyzed. Explicit reasons, which may be the genesis of common misconceptions, have been outlined. The study can help students to revisit those concepts where they have understanding difficulties and sensitize practicing teachers to frame better curricula and pedagogies for teaching undergraduate and postgraduate quantum mechanics. This is highly relevant in the current context of constructivist approach to physics education.

Keywords: Alternative conceptions, Quantum mechanics

Introduction

"Effective teaching does not simply teach students what is correct – it also insures that students do not believe what is incorrect" (Styer, 1996). In this context, study of conceptual issues of quantum mechanics has become an important concern of physics education (Müller & Wiesner, 2002; Singh, 2001, 2008; Styer, 1996). Unlike classical mechanics, the realm of quantum physics deals with phenomena not directly observable in everyday experience. In classical mechanics, misconceptions/alternative conceptions mostly arise from everyday observations and generalizations made by the students. However, the origin of misconceptions/alternative conceptions in quantum mechanics may be attributed to mental processes because such concepts are far from everyday observations and experiences. It is observed that teachers also possess these conceptual difficulties. Empirical observations suggest that these difficulties are easy to slip into. Hence, sometimes it may be just sufficient to identify the existence of such misconceptions. Such identification can sensitize the practicing teachers to some of the difficulties students are likely to face. Moreover, analysis and diagnosis of such difficulties can prove to be a valuable tool to design the course curriculum in quantum mechanics for postgraduate and undergraduate students.

In recent years, there have been a number of investigations of students' difficulties in understanding quantum mechanics (Müller & Wiesner, 2002; Singh, 2001, 2008). Styer (1996) has documented several common misconceptions regarding quantum mechanics. Some studies have also been conducted on the structure of basic quantum mechanics course (Müller & Wiesner, 2002). Most of these works have been carried out on the students of American (Singh, 2001, 2008) and German Universities (Müller & Wiesner, 2002). However, no such elaborate study seems to have been carried out on undergraduate/postgraduate students of Indian universities/ colleges, where quantum mechanics is an essential ingredient in the curricula of physics. Studies conducted outside have mainly focused on advanced level quantum mechanics and most of the students' misconceptions have been diagnosed through interviews (Müller & Wiesner, 2002; Singh, 2001, 2008). In the present study we have tried to diagnose the understanding of undergraduate and postgraduate students of a few Indian Universities and Institutes apart from some teachers, in the domain of introductory quantum mechanics. In the following sections we present the test design followed by the analysis of question-wise responses, and conclusions. We expect the findings will help students and teachers in this important domain of study.

Test Design

The diagnostic test comprises of 5 questions on basic aspects of quantum mechanics. Before administration they were ratified by the physics faculty members of National Institute of Science Education and Research (NISER), Bhubaneswar and Utkal University, Bhubaneswar. The first two questions have been structured following the list of Styer (1996) and the rest are designed to probe the understanding of the three fundamental equations of quantum mechanics. The test was administered to 47 postgraduate students (23 from Utkal University and 24 from Pondicherry Central University), 12 undergraduate students from NISER, and 15 teachers who were attending a Refresher Course in Physics at Institute of Physics (IoP), Bhubaneswar. We have also taken interviews of 7 postgraduate students of Utkal University and 8 undergraduate students of NISER. Our sample population includes undergraduate students of a national institute, who have taken two courses of quantum mechanics, and postgraduate students from a state university and a central university who have taken three courses of quantum mechanics (including one during their graduation). The participating teachers have 5-10 years experience in teaching quantum mechanics. Maximum time given to answer the questions was one hour and time for interview was not specified. The test was conducted in a group situation whereas interviews were conducted individually. Each interview was structured around why a respondent gave a particular answer to a test item and the follow-up questions. The interviews were recorded and later transcribed.

Analysis of Responses

The responses, both in written and oral forms, have been analyzed question-wise below. Wherever possible, the present findings have been compared with the earlier findings, available in the literature. We have tried to find the genesis of misconceptions which may be due to the pedagogy, respondents' background, curricula, textbooks and institutions.

Q1. What is the dimension of a wave function?

In our written diagnostic test, 7 out of 12 undergraduate students of NISER (58%) wrongly recognized wave function as dimensionless whereas 14 out of 23 students of Utkal University (61%) recognized it as having dimension L-3/2. Surprisingly 3 out of 15 teachers (20%) believe that wave function is dimensionless and 7 teachers (50%) believe it to have dimension of length. However, in the interviews, the dimension attributed to wave function varies from dimensionless to LT, L-1/2, L-3/2, and L-n/2 (where n means n-dimensional space) etc. One typical reason given by the student for assigning length (L) and time (T) as the dimension of the wave function is, "Wave function is represented by ψ (x, t), so it is space and time dependent and hence dimension is L and T". Interestingly, the student who gave dimension of wave function as L-1/2 argued his answer in this way, "Since ψ (x) = (2/L) 1/2 sin (n π x/L), ψ (x)*ψ (x) = 1/L. So ψ (x) has dimension of L-1/2". The reason for dimension of wave function to be L-3/2 was worked out by a student during interview in the following way: $\int\psi^*\psi\, d^3r = 1$, $\psi|N|^2|\psi|^2\int d^3r=1$, $|N|^2|\psi|^2=1/V=L^{-3}$ so $\psi = L^{-3/2}$. A student who claimed that wave function is dimensionless argued this way, "ψ has no physical interpretation, but probability associated with the wave function has some meaning and probability is dimensionless, so wave function is dimensionless". As an anticipated alternative conception, maximum number of students gave the statement: "$|\psi(x)|^2$ gives the probability of finding a particle at a point x". With the help of some guided questions we could convince them that $|\psi(x)|^2 \Delta x$ is the probability of finding the particle within an interval Δx around x. So, in the limit $\Delta x \to 0$, $|\psi(x)|^2 \Delta x = 0$, which means probability of finding a particle at a definite location is vanishingly small.

Many students were found to keep on contradicting their own statements/ideas on probability, sum of all probabilities, integration over all probabilities, etc. One example of such a contradiction was the situation when the same student made three different statements like, "Dimension of the wave function is $L^{-1/2}$; $|\psi(x)|^2$ is the probability of finding the particle at any particular position x; probability is dimensionless". The plight of another student to reason out his own statement, "The integral over all probabilities must be 1", was observed when he admitted that probability is discrete. It is clear that most of the students have not distinguished the fact that unlike classical probability quantum probability is a distribution.

On being asked about the dimension of a wave function of a system of two non-interacting particles in 3 dimensions the student answered that it would be L^{-6}. He missed the link that the wave function $\psi(r_1, r_2)$ is now in the product space $\psi(r_1)\,\psi(r_2)$ and in product space, dimensions of wave functions add up. So the dimension would be L^{-3}.

Q2. The probability density $|\psi(x)|^2$ completely specifies the quantum state ψ (x). Give your argument in favor or against this.

In our diagnostic written test 75% undergraduate students of NISER and 13% students each from Utkal and Pondicherry universities and 40% teachers gave the right answer but without any justification. Similarly, during interview also 41% students gave the right answer but with incorrect justifications. The following examples are typical: "It does not give a deterministic value, it gives only probabilistic value". "No, because the quantum state of ψ (x) may be complex but the probability is always real". "No, it does not specify energy". On being asked what is not specified in $|\psi(x)|^2$ one student answered, "Eigen value, amplitude". This shows the lack of realization of the significance of phase in a wave function. In most of the quantum measurement problems usually dealt with in basic quantum mechanics course, phase does not matter. One student was asked to compare $|\psi(x)|^2$ for two different stationary states, one with real and another with complex energy and then to identify which state corresponds to decaying state. At first, he was reluctant to accept energy as complex, but later could understand that the imaginary part of energy plays the role of phase and is indispensable to describing a decaying state.

The student who argued in favor of the statement in the question said, "$|\psi(x)|^2$ specifies completely the probability of finding the particle in quantum state ψ (x) because it does not depend on time". On being asked to be precise, she said, "$\int|\psi(x)|^2 dx$ is always one". A good remedial to this misconception is to assign an exercise calculating the expectation values for momenta of two Gaussian wave packets (Ghatak & Loknathan, 2007; Styer, 1996), say $\psi_1 = (1/\Pi\,\sigma^2)^{1/4} \exp(-x^2/2\sigma^2 + ip_1x/\hbar)$ and $\psi_2 = (1\Pi\,\sigma^2)^{1/4} \exp(-x^2/2\,\sigma^2 + ip_2x/\hbar)$ with identical probability densities but different phases, and hence with different expected momenta p_1 and p_2. Such problems that provide meaningful conceptual reinforcement are desirable for better understanding.

Q3. What do you understand from the equation $\hat{H}\psi=E\psi$?

This short form equation carries a lot of quantum description. So, we were interested to probe the alternative conceptions of the students on this equation. In our written diagnostic test only 31 % students of Utkal University and 33 % teachers have rightly identified it as an eigen value equation. But maximum percentage of undergraduate and postgraduate students in the written test and interview presented the same usual statement, "When operates on ψ it gives the eigen value E and the same state ψ". According to a recent study conducted by Singh (2008) 11% graduate students of United States believe that any statement involving a Hamiltonian operator acting on a state is a statement about the measurement of energy. To compare the ideas of Indian students with those of American students on this issue we asked the students whether this statement refers to measurement of energy. Sixty eight percent of all the students denied it and others were not even aware of such measurement. A typical denial goes like this, "It is a mathematical formalism and if it were an energy measurement operation, then eigen function would have collapsed to a different state". We intended the respondents to recognize that Hamiltonian acting on any state of the system Ψ will give the same state back only when ψ is stationery i.e. when Hamiltonian is independent of time. Interestingly, 20% teachers have interpreted the equation $\hat{H}\,\psi=E\psi$ as conservation of energy and the same view is expressed by 32% NISER students in interviews. We expect them to understand that although energy is conserved, $\hat{H}\,\psi=E\psi$ need not be always true even if the Hamiltonian is time independent. For example, if ψ is a linear superposition of stationery states, $\hat{H}\,\psi = E\psi$ although energy is conserved (Kuila, 2008; Singh, 2008). In the written test one student wrote, "The particle describing wave like motion is given by ψ and has energy E". Another student identified the equation as Schrodinger's equation but could not recognize it as time-independent. When asked about the characteristics of stationary state he referred to it as a quantum state of constant energy but could not mention any other attributes of the stationary state.

Q4. What is the meaning of Δ E and Δ t in the equation of uncertainty relation $\Delta E.\Delta t \geq \hbar/2$?

Eighty four percent students during interview uttered the same sentence to describe the time-energy uncertainty relation, "If we measure time precisely we cannot measure energy precisely". It is clear that students have understood this relation very much in line with position–momentum uncertainty relation. One student said, "x and p_x are canonical conjugate so also E and t, hence both the relations are analogous". On being asked whether time is an operator and how time has entered into quantum description, a student replied, "Time has entered through the energy operator $i\hbar\,(d/dt)$". It may be concluded that students do not realize that observables are represented as Hermitian operators and time entered quantum description as a parameter. Even if time-energy uncertainty relation looks analogous to position-momentum uncertainty relation the former cannot be interpreted in line with the latter. Eighty three percent undergraduate students of NISER, 46% postgraduate students and 53% teachers in written test identify ΔE and Δt as uncertainties in measurement of energy and time. During interview we asked, "If we look at a hydrogen atom for a very brief period of time, say a µsec, then what will happen to its sharp transition lines?" One student promptly said, "Δ E will become broad and it will look continuous". We intended the students to realize that the relation $\Delta E.\Delta t \geq \hbar/2$ implies, if the dynamical state exists only for a characteristic time of order Δt, then the energy of the state cannot be defined (Bransden & Joachain, 2004) to a precision better than $\hbar\,\Delta$ t. Interestingly, one NISER student identified it as an interpretation of conservation of energy. He argued, "We have calculated pion mass using the relation $\Delta E.\Delta t \geq \hbar/2$ which preserves the conservation of energy". The students ought to realize that one of the interpretations of $\Delta E.\Delta t \geq \hbar/2$ is that the principle of conservation of energy may be violated by an amount ΔE for a time interval Δt, related through the uncertainty principle. This interpretation helps in understanding the generation of virtual particles in quantum mechanical processes.

Q5. What is the condition for the de Broglie description of a quantum particle?

In our written test only 25% undergraduate students of NISER and 8% postgraduate students have written the condition correctly. Others have just realized it as an equation for wave-particle duality. However, our intention is to probe the understanding of students on the various ideas hidden in this simple equation. On being asked to describe the situation when p = 0, the students promptly answered, "λ will be ∞". But they could not explain it physically. One NISER student replied, "p = 0 means particle at rest, so E = 0 but this state is not allowed in quantum mechanics". To probe in detail the students' understanding on the consequence of definite momentum we asked them to work out the probability density of finding a particle which is in a definite momentum eigen state. One student rightly derived the wave function as $\hbar\,(x) = C\,e^{(ipx\,\hbar)}$ and $|\psi(x)|^2 = |C|^2$. But, even then, he could not interpret the result that this shows uniform distribution which means the particle is likely to be found everywhere. Students have failed to observe that uncertainty relation is inbuilt in the de Broglie relation which connects a particle of definite momentum to a monochromatic wave of wavelength λ. They need to realize that a monochromatic wave, expressed mathematically as exp [i(2πx/λ-ϖ t)], means that such a wave exists over all space at a given instant of time (Kuila, 2008). This implies position uncertainty of a particle with a definite momentum. They can easily work out that the velocity of a de Broglie wave is greater than c in violation of the principle of relativity (Kuila, 2008). This has in fact led to the concept of wave packet as a quantum representation of a particle instead of a monochromatic wave as suggested by the de Broglie relation. An appropriate remedy for this concept could be to prescribe a Gaussian wave packet (Ghatak & Loknathan, 2007) to the students, ask them to find Δx, use Fourier transform to find Δk and using the condition of Fourier transform, $\Delta x. \Delta k \sim 1$, establish the uncertainty relation.

Conclusion

After analyzing the responses obtained from the diagnostic written test and interviews we arrive at the following conclusions. The reason for the existing alternative conceptions of undergraduate students of NISER pertaining to Q1 and Q2 may be traced to the fact that their syllabus starts from Schrodinger's equation. So they are not so comfortable with the concepts of wave function, probability density, dimension of wave function etc. which normally precede Schrodinger's equation. About other students it is found that incorrect overgeneralization of the concept of classical probability and inability to distinguish two closely related concepts like probability and probability density have led to more and more confusions. Moreover, phase dependent phenomena such as scattering, decaying particle etc. are usually not dealt with in basic quantum mechanics course. So students feel probability density is enough to specify the quantum states. In Q3 the students' inability to identify and interpret the various concepts like eigen function, eigen value, stationary states, discrete eigen values etc. hidden in the mathematical expression was clearly observed. While interpreting this equation during classroom teaching elaborate discussion by the teacher is desirable. In Q4 lack of visualizing an atomic transition, failure to distinguish between two closely related concepts in the position-momentum and energy-time uncertainty relations and difficulties to interpret individual parameters (ΔE and Δt) are noticed. Inability to link the concept of operator formalism learned earlier to the present concept for recognizing time as a parameter is also observed. Often textbooks tend to discuss more on position-momentum uncertainty relation than on time-energy uncertainty relation. In Q5 skill of interpreting mathematical relation by varying its parameters and drawing qualitative inferences by the students is lacking. Overlooking the conditions for certain description was noticed in this case.

Our findings can definitely help the practicing teachers realize the probable shortcomings of the students even before the class. This can also guide them to frame better curricula and pedagogies for teaching undergraduate and postgraduate quantum mechanics. Moreover, this study can help teachers to prepare themselves to address such alternative conceptions and thereby enhance their competency. With such written diagnostic tests and interviews students become aware of their conceptual difficulties which otherwise they would continue to carry. The think-aloud protocol used in interview and open-ended questionnaire help them reflect upon their own difficulties. This is a part of critical pedagogy recognized as highly relevant in constructivist approach to teaching learning. In fact, the undergraduate students of NISER suggested that this type of diagnostic test, both in written and interview mode, is highly desirable after each unit of their syllabus to address their alternative concepts. They admitted that this interaction has motivated them to revisit the above discussed concepts.

Recently, to provide conceptual framework to strange and counterintuitive phenomena the concept of virtual laboratory (Müller & Wiesner, 2002) and quantum interactive learning tutorial (Singh, 2001; Singh 2008) have been suggested. It may take some time to realize quantum virtual laboratory in Indian classrooms but interactive learning tutorials can be a reality soon. This study can motivate our Indian students and teachers in these directions also.

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