ASSISTING LEARNERS TO CONSTRUCT UNDERSTANDING

Satyawati Rawool

P.V.D.T. College of Education for Women, Mumbai, India

rsatya.rawool@gmail.com

Helping student teachers to transform their concepts of learning, teaching and evaluation demand investment of intellectual and emotional energies on part of facilitator. The efforts put into evolving learner centered culture helped me to gain insight about my own modes and ways of communication and learners' ways of constructing their understanding. How communities of learners work toward achieving intersubjectivity (identical state of cognitive experience) with respect to mathematical knowledge construction (Cobb, et. al., 1991) is a useful question that helps one to think about developing learning activities for pre service teachers.

Keywords: Intersubjectivity, Radical constructivist approach

THEORETICAL FRAMEWORK

Psychological basis of designing learning situation

According to von Glasersfeld (1991, 1995), the first order model of knowledge construction is composed of mathematical knowledge of a facilitator and second order model is composed of interpretations done by facilitators as they observe personal knowledge construction activities undertaken by learners. There is no simple transformation of the knowledge of facilitators that can be used for understanding learners' process of knowledge construction. It is necessary to establish and analyse possibilities for interactive mathematical communication between facilitator and learner and also among learners. For this to happen a facilitator need to be aware about how she is interacting with learners and its impact on the learners' construction of mathematical understanding.

The second order knowledge construction helps a person to go beyond her or his own field of experience and reach into that of others. This has a very important role to play in the stabilization and solidification of ones experiential reality. This helps all members of a learning community to create an intersubjective level through which they are lead to believe that concepts, schemes of actions, goals, and feelings can be shared by all. This is the kind of intersubjective level at which we feel that we have a common and identical understanding and we think (or feel) that it must be more real than personal understanding.

In order to practice this insight in day to day facilitation, a facilitator needs to consider her personal frame of reference and learners' frame of reference simultaneously. This allows the facilitator to be a second order observer and operate as external to the situation in which she is present. Thus the facilitator as a person is an observer of her or his own situation of work (Maturana, 1978).

Philosophical basis of facilitator's work

"Fallibilist" philosophies of mathematics are gaining ground for last few decades. This philosophy views mathematics as human, corrigible, historical and changing (Davis & Hersh, 1980; Ernest, 1994a, 1994b; Lakatos, 1976; Tymoczko, 1986). This view of mathematics considers mathematics to be a product of social processes and therefore in terms of concepts and proofs, mathematical knowledge is open to revision. This view has changed the practices of mathematicians, their perceptions about its history, its applications and the role played by mathematics in human culture. The "Fallibilist" do give due importance to the logical rigour of mathematics and its unique, fixed and permanently enduring hierarchical structure. But this view also accepts that mathematics is composed of many overlapping structures. This is evident through many studies related to ethno-mathematics. They prove that different communities have developed mathematics needed to them. Similarly many people, who did not study mathematics in school, evolve their own personal mathematics that serves their personal purpose. This suggests that learners should be given the opportunity to construct their own mathematics.

Learner centred learning environment

According to Duckworth (2006), a facilitator has the following responsibilities; providing some accessible entry point, presenting subject matter from different angles, eliciting different responses from different learners, opening a variety of paths for exploration, engendering conflicts and providing surprise, motivating one to open beyond oneself, helping learners to realize that there are many other points of view yet to be uncovered and that aspects of reality are not exhausted and thus any person exploring the subject can have new thoughts about it.

Learners bring their prior expectations, interests and knowledge to any learning experience. My learners were adults and most of them joined course just to get a teaching certificate with excellent grades. Of twenty one students, four were mathematics graduates while others were science graduates. Efforts were made to make them aware of their social responsibilities and philosophical urges in the beginning of the course. Learners need motivation to open up to experience many fascinated aspects of ordinary world, they should experience and feel that their ideas have important place in their and others' learning. My learners were in the habit of listening or reading ideas, reasoning, interpretations developed by teachers and remembering them for writing in final examination. They and their teachers never felt the need for comprehending information for applying it in various situations in their own life. They felt that meaning making is a waste of time.

Learners have a potential to have wonderful ideas (Duckworth, 2006) and this affects their learning significantly. My learners did not believe that they can have their own ideas and explore them for constructing deeper understanding. Opportunities were created to help them realize that as learners of any age and background they too have many questions, doubts and ideas but which perhaps during their school learning they did not get a chance to make public, have fun with and learn useful concepts and processes. Learners need something complex that challenges them to explore. They should get a chance to experience internal conflict with respect to the subject matter. In my case in the beginning, learners did not know that they can enjoy intellectually taxing work. But slowly they started enjoying the learning process and started believing that they are being transformed.

Initially I did spend lot of time with learners encouraging them to talk about their school learning experiences related to mathematics to put forward their ideas about education, their image of ideal teacher and ideal learner, learning of various subjects, relevance of learning mathematics, its relevance in their personal life, their personal goal of development, and evolve a personal mission statement. Discussion on these issues helped learners to experience that they can think seriously on personal and social problems. Many students expressed that, for the first time they tried to talk about something that is important but is not to be remembered only for examination.

Transcript of a Learning Situation

Here is a brief account of a sample learning situation.

To evolve this type discussion constant encouragement was needed. To help them to involve in critical and analytical thinking, many encountering interruptions were necessary. Student teachers required a lot of time to think about the concerned question.

A group of student teachers was struggling to design a work sheet for assisting pupils to construct their own understanding of operations on fractions. When a sample worksheet was ready they were asked to play a role of pupils of age group10+ to test its effectiveness with respect to listed learning opportunities that were offered to the learners. All students were given time to discuss in small groups, the concerned question each time and then large group discussion was conducted.

They began adding pairs of fraction like given below.

$\frac{1}{6}+\frac{3}{6}$ $\frac{3}{7}+\frac{4}{7}$ $\frac{4}{9}+\frac{7}{9}$ Write addition and justify the process.

To facilitate pupils' understanding the group decided to make learning activity concrete by giving figural representations that is divided areas and shaded parts. Each fraction was represented by separate figures on the work sheet. At this juncture I, who was playing a role of pupil, completed this work as shown below.

$\frac{1}{6}+\frac{3}{6}=\frac{4}{12}$

I wrote following justification. Out of one rectangle I have selected one part from one set of equal parts and from another set three parts. Thus from available twelve from available twelve equal parts, four parts are selected.

With this I tried to create an entry point for student teachers to construct or reconstruct or modify their knowledge of mathematics and mathematics teaching. They were asked to figure out the problem and get a solution.

All of them suggested that only one rectangle with six equal parts should be provided along with exercise to avoid confusion. The following discussion took place after this.

Facilitator: Will this trick of giving one figure, solve the problem of understanding of addition of fractions? How will you explain the fraction 8/6?

Student: But is it not a correct way of thinking about addition of fraction? We felt the problem when you said that it is a problem. What will we be doing as teachers if we don't know correct mathematics? Your justification appears logical to me. I was adding as per the rules given to us by our teachers. You shouldn't add denominator. I did not even try your way of doing it.

Facilitator: Why should I accept rules without understanding its logical base? It was my way of making meaning of addition of fractions. Addition is not a new process to me. So when one sixth part of each of the five different fruits are added, I will get 5/30 as sum. Isn't it?

Student: I find your logic correct. If in case pupils are giving this type of justification then logically; I need to accept it. We learn mathematics as a set of rules to be remembered and to be followed rigorously. But I think that we must give pupil the freedom to have their own logic.

Facilitator: Yes…but we need to establish logical consistency…Are you sure that additions completed by you are mathematically correct…consistent?

Student: Yes. They are correct according to whatever we have learned…

Facilitator: Then why don't you work out that logic deductively? (After some time…)

Student: It's beyond our capacity to think like this. It never happened in our school class. We all were getting correct answers… or might be that some pupils were getting wrong answers and they were ignored by our teachers… our teacher asked some of us to write it on the board and narrate the procedure of getting answers for the benefit of other students…It was an award for us. We used to write solution for other student to show our benevolence.

Facilitator: Goal of your teacher was to complete the portion and see that all learners get correct answers. You had no problem till graduation. You are all merit holders and now you are in the world of work. Isn't it? Now your responsibility is to facilitate development of learners' overall personality and assist each one of them to develop their skills of developing understanding different processes is your goal.

Student: We are not aware of all these things. We did read mathematics book because you wanted us to analyse every aspect of it and design activities. This helped us to understand place of mathematics in personality development.

Facilitator: No problem. Now let's try to understand this operation as a novice. This will help you to be empathetic with your learners. Let's go step by step. Let's try to make meaning of the fraction one upon six.

Student: Any thing is divided into six equal parts and we consider only one part out of it.

Facilitator: That means to begin with we have something and we are considering it as a whole. Is my thinking is relevant? How do we describe this "whole and part relationship?"

Student: The whole is divided into equal parts…the whole is considered to be one… and that one is divided equally into as many parts that we want.

Facilitator: How does this relationship helps us to understand the process of addition?

Student: Equal parts are added therefore no need to change denominator.

Facilitator: Will you please elaborate your understanding…? I do not understand what you mean by addition of equal parts…no need to change denominator…

Student: I did not think on this yet. It was a guess. In our school class we used to guess answer and some times by chance we were correct.

Facilitator: Well. I am here to give you a chance to understand…and I think you will not be having any problem with addition of whole numbers?

Student: No… not at all.

Facilitator: Then why not apply same logic in this case.

Student: How can we apply same logic in this case?

Facilitator: Let me try to revisit the process of addition of whole numbers. What do I do when I add five books? How do I find out that these are five books?

$1 + 1 + 1 + 1 + 1 = 5$

What I think when I consider numbers…numerals and not objects…only…? They are wholes… and I assume that they are equal wholes… Is it necessary to have equal wholes? I need to think about it.

Now let me think about this addition…

$\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=\frac{5}{5}$

In this case I am assuming that my "whole" is made up of five equal parts. As per this assumption value of each part is one upon five. So I must get one whole by the process of addition of five parts. Value of denominator indicates the number of equal parts into which we have divided our whole. Value in the numerator indicates number of parts added together, each having value one upon five. Thus I am getting addition five out of five equal parts that is equal to a whole mathematically.

Student: But in this case you are thinking about one…single whole. What if you are considering that these parts are from two different wholes?

Facilitator: Why not? They can be one fifth part of five different wholes? Try to explain this addition to yourself.

Student: Yes. I have five wholes. Do I need to have equal; I mean identical wholes?

Facilitator: You are explaining the situation to yourself…you decide about assumptions… and they should be logically consistent.

Student: If I havE five different fruits and cut each one into five equal parts…it doesn't matter…Let me have five identical wholes…And if I am dividing each whole into five equal parts then the value of each part is one upon five with respect to the whole. It is immaterial whether they are from single whole or not. Thus five parts, of one fifth value each; equals a whole mathematically.

Student: Can I try to explain multiplication also? Let me try…

$\frac{2}{3} \times \frac{8}{5} = \frac{16}{15}$ …this is difficult to understand…explain…

I will take a simpler multi plication to begin with.

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ It's different than whole numbers. The value of a product is less than the multiplicand and the multiplier. I never observed this situation any time before.

Can I make use of any other product to explain this?

Facilitator: It's up to you to decide…we must make things clear to ourselves.

Student: If I decide to multiply two by one half, what do I get?

$\frac{2}{1} \times \frac{1}{2} = \frac{2}{2}$ Verbally it is half times two…and…it is one… that is one. Two one half means a one whole. Simi larly one multiplied by one half will be equal to half of one…that is one half…

$\frac{1}{1} \times \frac{1}{2} = \frac{1}{2}$ This is the representation of that multiplication.

Student: But why are you writing two as two upon one?

Student: It's very simple…we can write all numbers in that form.

Facilitator: Do you mean to say that we have the liberty of writing it that way… or do we have any mathematical logic to explain it?... or can we do without writing it that way?

Student: It is not necessary to write the whole number in the form of fractional number…I mean numerator upon denominator… Here logically we consider that whole is divided into whole…or it is not divided into parts. We write one in the denominator to indicate that whole takes part in the process as a whole. We are not dividing it… Now I understand how a set of whole numbers was defined as a rational number…

Facilitator: Learning of any skill, concept or process actually involves this kind of thought experiments. Please carry on your experiment…

Student: I got it now. When I multiply any number by one half; I am reducing it to half…it is half times and that is why the product is always less than multiplicand and multiplier. It's really interesting. I never looked at these processes any time before this way and graduated without knowing it.

Facilitator: This happens because you were not aware of meaning of learning. Our responsibility is helping pupils to learn learning skills. Every learner should be helped to learn different skills and should be given sufficient time to verbalize their thinking.

Student: But I think that pupils will not be able to verbalize any explanation… they will not be able understand it. We could do that because we are grown ups…

Student: and each one will come with different explanations then what will we do as facilitators?

Facilitator: Yes…That what we expect to happen. They should learn to accept the fact that they can think, they can think on their own, there is nothing wrong if there is any logical gap or inconsistency; when one is aware of logical gap it can be repaired by thinking logically. Then only we will come to know about their experiences and their way of making meaning.

Student: I think that we were also getting correct answers mechanically and not logically. Now I am experiencing real learning.

Facilitator: What will happen if you accept the logic that I proposed?

Student: Every one will get different sum. For example, if I am giving ten rectangular areas, each divided into five equal parts, and in all I shade five parts and then ask you to add shaded parts you will get 5/50 as an answer. It indicates that you are not adding value one upon five…five times.

Student: You were not adding values of parts. You were looking at the shaded parts, counting total number of parts, counting number of shaded parts and writing ratio of number shaded parts to that of total number of parts.

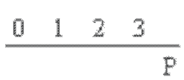
Student: I will design one activity to understand learners thinking. I will draw more that required figures divided into equal number of parts. Now let us design a challenging learning activity.

Sample activity is given below:

State whether each of the statement is logically correct or not along with justification in your note book and discuss it with your peer groups. Work in a group of three. You need to experiment with many fractions and then only finalize your answer. Let's test our understanding of concept "fractions".

What is the meaning of fraction 9/3?

- It means that in this case the whole is composed of three equal parts or distinct objects. Nine such equal parts are considered.
- Nine equal wholes are divided into three groups of equal number of wholes.
- It's not necessary here to assume to have nine equal wholes and then divide each one into three equal wholes.
- It is the addition of nine parts that are equal and each has the value "one upon three".
- We can use this number to indicate coordinate of point "P" as shown on the following number line.



- This number can be used to represent three objects.
- This is a number and it can be used to indicate ratio of ages of nine years old to that of three years old.
- There are nine equal whole, each whole is divided into three equal parts and one part from every whole is considered together.

Result

As this learning activity progressed student teachers were requested to reflect on their learning of mathematics at school level. This activity proved to be an entry point for them to experience authentic learning. They were motivated to speak about their emotional states as well as their intellectual struggles related to understanding of fractions. They also motivated to think about designing mathematical tasks that did not demand mere drill work. When they used these types of work sheets in the class of more than eighty students, they found that students were working together without any problem of discipline. Many pupils expressed their interest in doing this kind of activity. Student teachers also experienced difference between teaching for imposing a concept or process and facilitating for assisting learners to construct their own knowledge. Creating scope for learners to articulate their thinking in a class of more than sixty pupils was a challenge for them. They started engaging themselves in understanding mathematical concepts and also realized that there is hardly any end to such type of learning.

References

Cobb, P., Wood, T., Yakel, E., Nicholls, J., Wheatley, G., Trigatti, B., & Perlwitz, M. (1991). Assessment of a problem-centered second-grade mathematics project. *Journal for Research in Mathematical Education*.

Davis, P.J., & Hersh, R. (1980). *The Mathematical experience*. Harmondsworth: Penguin.

Duckworth, E. (2006). *The having of wonderful ideas and other essays on teaching and learning*, (III edition) New York: Teachers College Press.

Ernest, P. (Ed). (1994a). *Constructing mathematical knowledge: Epistemelogy and mathematics education*. London: Falmer Press.

Ernest, P. (Ed). (1994b). *Mathematics, Education & Philosophy: An International perspective*. London: Falmer Press.

Lakatos, I. (1976). *Proofs & Refutations*. Cambridge: Cambridge University Press.

Maturana, H.R. (1978). Cognition. In Hejl, Peter, M., Wolfram, K. Kock and Gerhard Roth (Eds.), Wahrenhmung und Kommunikation. Frankfurt: Peter Lang, 1978.

Tymoczko, T. (1986). Challenging foundations. In T. Tymoczko (Ed.), New Directions in the philosophy of mathematics. Boston: Birkhauser.

von Glasersfeld, E. (1991). *Radical Constructivism in Mathematics education*. New York: Kluwer Academic Publisher.

von Glasersfeld, E. (1995). *Radical Constructivism: A way of knowing & learning*. London: Falmer Press.