The Social Semiotics of Mathematics Classroom Discourse: Developing Teacher Capacity to Explore Non-Routine Problems in Urban Classrooms

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This paper reports on a research project exploring the social semiotics of mathematics instruction in New York City middle schools. Participating teachers attended a Lesson Study Group and developed their skills at decoding the linguistic and diagrammatic challenges of orchestrating whole-class conversations about non-routine problems in urban schools. This paper focuses on how the teachers experimented with one such problem in their own classrooms.

Keywords: Discourse, Semiotics, Diagrams, Problem solving, Mathematics instruction

Introduction

This paper reports on a research project on the social semiotics of whole-class interaction in mathematics classrooms. The project engaged 12 middle school mathematics teachers who worked in urban "high needs" schools in New York City. The term "high needs" is used by the city to designate schools that serve communities with high poverty and whose students qualify for free or reduced price lunches. The schools in this study also serve highly diverse communities in terms of linguistic, cultural, ethnic and racial differences. The project aims to enhance teacher capacity to negotiate and facilitate student code-switching between the highly formal language of mathematics and the everyday language of students in these diverse school settings. Participating teachers meet six times per semester in a lesson study group to collaboratively work on developing their understanding of the social semiotic challenges of teaching and learning mathematics in such schools. In this case, social semiotics is defined as a framework which focuses on the function of multiple semiotic systems (symbolic notation, oral and written language, graphs and visual displays, gestures and the use of material objects) and grammatical patterns (technical vocabulary, dense noun phrases, "being" and "having" verbs, logical conjunctions, visual codes, canonical gestures) in spoken, written and performed mathematical texts. The "social" part of social semiotics indicates our commitment to theorizing "sign use" as an inherently socio-cultural practice (Morgan, 2006). Moreover, social semiotics brings a critical lens to the study of discourse in that instructional utterances and actions are always seen as embedded within the regulative discourse that structures power relations within society. This socio-cultural framing is crucial for studying the code-switching habits of students and teachers as they grapple with the distinct characteristics of school mathematics discourse in these urban contexts.

In this paper, we show how participating teachers increased their capacity to analyze facets of classroom discourse and to facilitate–within their own classrooms–student negotiation of the semiotic and linguistic aspects of non-routine problems. We define "non-routine problems" as those problems that are non-procedural and new or unfamiliar to students. Our focus on the use of non-routine problems in high poverty urban classrooms is meant to counter the dominant focus on procedural mastery and symbolic manipulation in such contexts. We believe that the focus on procedural mastery in these contexts functions to further mystify the mathematics register as an esoteric textual ritual. Using rich non-routine problems in these schools is crucial if we are to interrupt pedagogies of oppression that re-inscribe socio-economic inequity. Although we are interested in precisely how to increase access to the cultural capital of non-routine problem solving, we are careful to distinguish the use of non-routine problems from a constructivist philosophy that assumes middle-class habits of learning for all students. We believe that mathematics teachers must make visible the codes for legitimate textual production so as to better equip their students with the skills for school success. We argue that increased attention to the small and significant linguistic and semiotic facets of classroom multi-modal discourse (both language use, diagramming and other semiotic resources) can help teachers better leverage non-routine problems, especially in those communities where access to the gate-keeping discourse of school mathematics has traditionally been denied. As Veel (1999) suggests, language and sign use are the source of "differentiated access to meaning potential... providing some students with the access to the technical meaning potential of mathematics while simultaneously denying access to others" (p. 206).

Like other lesson study groups, teachers in this study participated in a cycle of lesson design, implementation and reflection. Unlike other lesson study groups, our group focused on the linguistic and semiotic aspects of this process. Through this training, teachers developed skills at identifying particular linguistic, diagrammatic and gestural facets of mathematics discourse. They have also shown increased capacity to attend more carefully to these aspects while implementing lessons. In this paper, we follow teachers through their engagement with a non-routine problem entitled "The Fishpond Problem", discussing: (1) their attempt to solve the problem in multiple ways, (2) their analysis of its linguistic and diagrammatic challenges, (3) their analysis of a classroom transcript in which the same problem was used, and (4) their use of the problem in their own classrooms.

The Fishpond Problem

> *At every corner of a square fishpond there is a tree. Make the fishpond twice as big so that it remains a square but the trees remain where they are.*

One is immediately struck by the strange "reality" that is conjured in the fishpond problem: how can a fishpond be square? In the urban context of NYC, another more basic question might be: what is a fishpond? The reference to the "real" world of fishponds and trees creates an additional decoding task for students, and functions to inculcate students into the "myth" of the application of mathematics (Gellert & Jablonka, 2010). Research on "real world" word problems has shown that student inclination to decode a problem in terms of the rules of school mathematics and not in terms of their reality impacts hugely on their performance in standardized testing (Cooper, 1998a, 1998b, 2001). Studies of working class students in the UK indicated that they were more inclined to interpret word problems "realistically" and to thereby miss the coded mathematical meanings embedded in the text (Cooper & Dunne, 2004).

This raises many issues concerning how and to what extent any sort of "reality" should be cited or enlisted in application problems. In this paper, we address these issues from a new perspective by focusing on the way two teachers negotiated the contextual framing of the fishpond problem in their classroom after they had analyzed it through a social semiotic lens in the study group. We hope to show how attention to the linguistic and diagrammatic aspects of the "word problem genre" (Gerofsky, 2004) gives teachers a way to rethink the use of such problems.

Unpacking the Social Semiotics of the Problem

Bakker and Hoffmann (2005) use Peirce's tripartite theory of signs to define diagrams as "complex signs" composed of icons, indices and symbols. Icons have some physical resemblance to that which they signify, indices are the traces or imprint of the referent (often a deictic or pointing function), and symbols signify through custom or habit. The task of moving from the iconic to the symbolic in diagramming is thus a complex semiotic skill central to the doing of school mathematics. Indeed, such diagrammatic norms are essential in problem solving. Hoffmann (2005) argues that representational systems are normative in that initiates must submit to these symbolic norms: a mathematician "submits to the inference rules and conventions when experimenting with a diagram and these define the limits of possible transformation" (p. 49). He points out that one must internalize the normativeness of representational systems in order to experience this inevitableness.

When our teachers were given the problem and asked to work in groups of three, the first attempt at generating a diagram raised various semiotic issues, in that the trees were drawn as "icons" – that is, pictorially –instead of using points. Using iconic representations of trees introduced new problems and questions about the exactness of the trees location, orientation and dimensionality. The iconic or pictorial nature of the trees created the kind of "noise" that often inhibits problem engagement. And yet from a social semiotic perspective, noise of this sort is, in fact, the "real" that resists the normative conventions of mathematical diagramming. The teachers discussed how iconic figures, like the trees, often functioned, for their students, as familiar anchors in an otherwise unfamiliar picture. The juxtaposition of the iconic (trees) with the symbolic (square) creates a complex diagram that is less about representation and more about the creation of an imaginary world somewhere between the real and the ideal (O'Halloran, 2005). The mixing of sign systems in these complex diagrams is a social semiotic tool for negotiating the surreal scenario described in the word problem. In some sense, the trees are markers of language itself – they remind the student of the linguistic framing of the problem. They are also visual cues that carry ontological weight – they have more "being" or haeccity than a point, and if one keeps the trees as iconic trees, one may be less likely to forget the constraint that "the trees remain where they are". Teachers felt differently about how best to use this mix of iconic and symbolic signs in their classrooms, in each case preferring the approach that better reflected their own diagramming practices as problem solvers.

The teachers then moved to the task of doubling the fishpond. The first suggestion was to simply "expand" the square, but they noted that the trees would then be in the water. The facilitator introduced the "real" constraint that "Trees can't grow in water", thereby adding to the ontological weight of the iconic trees, and appealing to the myth of application to bolster the "reality" of the problem. This was a choice that was later discussed through a social semiotic lens, and the teachers were invited to critique this discursive move. The facilitator then asked the teachers to name the essential qualities of a square, and they stated "90 degree angles" and "Four equal sides". She then grounded the concept of invariance in the actual diagram they had created, asking, in a playful tone, "Does a square always have to be brown?" (brown was the color of the diagram), and "What else do you notice about this square that isn't an essential quality?", and then added "Does a square always have to sit on one of its edges?" It is important to note that the facilitator was modeling two important tools within a social semiotic approach to pedagogy – the first was explicitly directing teacher attention to the semiotic resource at hand, and asking them to "notice" (as opposed to "know"), and the second was the use of the material verb "sit" to animate the square and trigger thoughts of motion. This latter discursive move also models the need to go back-and-forth between material language (particular characteristics described using the material verb "to sit") and relational language (essential qualities described with the verb "to be"). At this point, one of the teachers (Lada) suggested tilting the square, and added the green lines in the Figure 1. Since we will see how Lada uses this problem in her class, it is worth noting that her intervention both tilted and doubled the square.

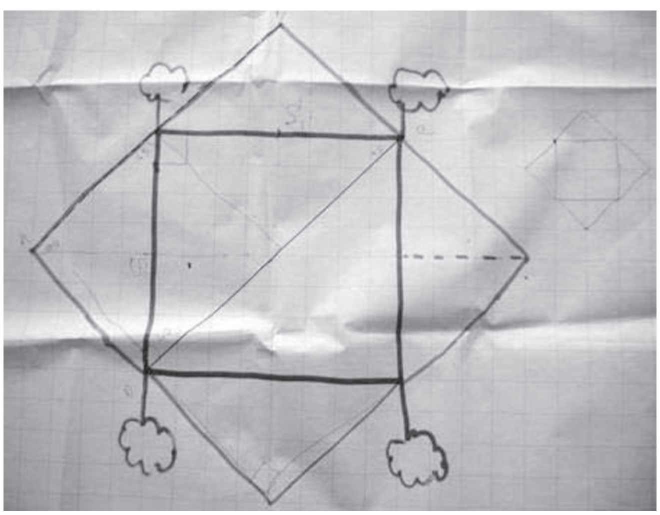

Figure 1: Lada's diagram

The teachers then worked in small groups generating explanations as to why this new green pond was twice the original pond. The ensuing discussion was a rich exploration of the geometric relations between the parts of the diagram. After exploring the problem, the teachers then discussed the limitations of the language used in the word problem. They were asked to consider two student responses to the original problem: (1) Let's dig the pond deeper and double its volume, and (2) Let's bend the elastic edges of the pond, drag the elastic pond between the trees, and then stretch it out and build the new one beside the original. These mathematically legitimate responses to the fishpond problem arise due to various kinds of ambiguity that are built into the problem. The first response is entirely legitimate because the problem has not specified what substance will be doubled, and so a student might propose doubling the volume. The second response is also an entirely legitimate topological response since one can bend the corners of the original square and drag it out between the trees and then stretch its border and enlarge it outside the original region.

These legitimate responses emerge because of two sources of ambiguity: (1) the lack of specific vocabulary about what should be doubled, and (2) the absence of a constraint that the pond should be built in the original region. These solution strategies, however, are not the solutions that the teacher is looking for. In other words, they are not sanctioned as legitimate (Cooper & Harries, 2010). For students who construct these divergent interpretations of the text, the issue emerges as to how they move through this sense of illegitimacy. In addition, how might the teacher validate these divergent interpretations while redirecting these students to engage the problem in the way she wishes, so that the particular mathematical content objectives can be explored?

To help them attend more carefully to the linguistic challenges of decoding the problem in a classroom, teachers were given a set of alternative versions of the fishpond problem. In each case, the alternative version addresses issues of word choice. Teachers were asked to consider the strengths and weaknesses of these alternatives:

- Imagine a square fishpond with a tree on each of its four corners. Use the space above to make the fishpond twice as big without changing the location of the trees. It should still be square. [The command "imagine" directs the students to the playfulness of the word problem genre]
- At every corner of a square fishpond there is a tree. Make the fishpond twice as big so that it remains a square but the trees remain where they are. [The word "at" instead of "on" construes the placement of the trees differently]
- Imagine a square fishpond with a tree at each of its four corners. The pond has a water source or spring at its center. Without moving the trees, make a new square fishpond that has a surface area twice as big and still contains the source for water. [The introduction of a specific quantity to be doubled (surface area) eliminates the possibility of digging the pond deeper]
- At every corner of a square fishpond there is a tree. At the center there is a water fountain. Enlarge the pond so that the square surface is twice as big, but leave the

fountain and the trees where they are. [The introduction of the constraint of a water fountain eliminates the possibility of dragging the pond and building it in another location]

— Four vertical pegs are attached to a board to form a square with a fifth peg in the center of the square. A rubber band is placed over the center peg (only). Without lifting the rubber band over any pegs, stretch the elastic to form a square whose area is twice that of the square formed by the original four pegs? [The reference to a material manipulative turns the probem into a material task]

— Imagine a square with a clearly marked center and four corners. Enlarge the square to twice the size while keeping the center and not enclosing the original corners. [The erasure of any reference to pond or other "reality" decontextualizes the problem]

— Given a square, create a new square twice the area of the original square and with the four original vertices on the new perimeter. [The language draws attention to the visual coincidence of the edge of the new square and the old vertex]

Teachers initially gravitated to the peg-version (No. 5), preferring it because they assumed that real pegs were involved. The facilitator asked them to treat No. 5 as though it were a word problem –that is, that there were no pegs, just written words. They then dismissed it as the least accessible. Bonnie's contribution below comes from this discussion, and reveals her awareness that version No. 5, with its emphasis on a material model outside of language, relies heavily on the presumption of a one-to-one mapping between the verbal and the physical world. Moreover, the original "square" in this version is an abstract relationship between the four pegs, and Bonnie points out how students would be challenged to connect the material metaphor of the elastic with the abstract metaphor of the square.

Bonnie: Um, well saying the rubber band is placed over the center peg so if I have like a peg and I have four other pegs or whatever and I'm putting something over the center peg, I'm just going to throw it over the center peg and like forget the original square, where's that? … So, I mean when it says you need to enlarge the square, it's like … what's the original square? … because I only put the rubber band over the center peg.

A discussion about the use of the word "on" in version no.1 versus "at" in version no.2 ensued. Teachers noted the ambiguity of "on" in version no.1, and wondered whether this minor but significant difference might help students better negotiate the use of iconic trees in the diagram. In multi-lingual classrooms, teachers need to attend rigorously to how these small words have multiple and different meanings in other languages.

From a social semiotic perspective, one of the most revealing discussions centered on the use of the word "imagine":

Cici: Um, I was just going to add, number one it starts off with the verb "imagine" and then it jumps to "use the space below" if I were to read it and start off with imagine I would have a picture in my head but I wouldn't think to draw a picture because it says "use the space below to make the fish pond twice as big" so it assumes that you already drew the first picture of the square that you were imagining.

Bonnie: I actually liked the imagine because the problem of saying like "when are we ever going to need this" or like "is this an actual problem" or "is this an actual fish pond that we're enlarging"—it, it makes it so that it is more of a story problem rather than…

In Cici's case, "imagine" meant "create a mental picture", and seemed to contradict the "use the space below". Cici suggests changing "imagine" to "draw", which is a direct command to perform a material act, and seems to rely on a strict boundary between thinking and doing. In Bonnie's case, "imagine" was preferred because it was code for the student to "make-believe" the situation and not question the reality of the described context – that being the absurd notion of a square fishpond. Bonnie and Cici are working with radically different meanings for "imagine", meanings that reveal (1) different assumptions about the relation between thinking and doing, (2) different assumptions about the role of the imagination as "image-making" versus "make-believe", and (3) different assumptions about their students' capacity to decode the command "imagine" in terms of its interpersonal positioning of the student within the re-contextualized discourse of school mathematics. Bonnie and Cici are also working in radically different schools. Cici, who is African American, works in a school that is primarily African American. Bonnie, who is white European American, works in a multi-lingual school primarily comprised of Russian and East Asian new immigrants.

Studying Teacher Discursive Moves

After exploring the alternative versions of the fishpond problem, teachers then discussed a transcript from a grade 8 classroom where the fishpond problem had been explored. A short excerpt is below:

T1: *Ok. Who can tell, in your own words, what this problem is about and what we have to do?*

S1: *There are four trees around a pond… a fish, a fishpond*

S2: *The pond is like a square*

T1: *Right. It's a square fishpond*

S3: *What's that?*

S4: *Never seen that!*

S5: *It's not a real pond. It's a made up one… sort of like a fountain. I saw one of those*

T1: *And what do we have to do?*

S6: *We have to do twice, two times the size of it…*

The teachers noted that the ambiguity of the problem was producing noise for the students. The facilitator asked them to consider how S1 starts off by simply noting that there are four trees around a pond, and that the statement is not contested by the class. The issue for the students, however, is the squareness of the pond, since they recognize immediately that squareness is not something you see in ponds – despite the brilliant effort of S5 to remind them of man-made "made up" ponds. Again, from a social semiotic perspective, what seems to be challenging the students is the conjunction of the mathematics register with the everyday discourse. In fact, a square pond highlights the disparity between the two, since it creates more dissonance than a square piece of bread or a square garden, both of which would be easier for American students to mesh in one image. The teacher discussion focused explicitly on how these two different codes were bumping up against each other. We will see that Bonnie will tap into this issue when she explores the fishpond problem a second time in year two of the project.

After reading through the entire transcript, the teachers decided that the central challenge in orchestrating a whole class conversation about this problem lay in how the teacher guided the students to tilt the square, without telling them to do it, and without showing them. The question of how, when and what to leverage in order to facilitate this problem solving strategy is a social semiotic question. Although it may seem less related to language or socio-cultural issues, the question remains a highly coded socio-cultural question about sign use. The challenge for social semiotics, and all socio-cultural theories of learning and identity, is to show how interactions at the micro-level, which seemingly pertain to "the mathematics itself", are indeed constituted and negotiated on a socio-cultural plane. Social semiotics encompasses all sign-use and argues that "visual modality rests on culturally and historically determined standards of what is real and what is not, and not on the objective correspondence of the visual image to a reality defined independently" (Kress & Van Leeuwen, 1990, p. 52). One can see in the classroom experiments that the teachers' discursive moves to facilitate the rotation of the square reflect a complex matrix of semiotic habits and cultural performance. Indeed, even the choice to focus on this transformation as the central strategy points to particular biases about problem solving (for instance, as opposed to a more open ended problem exploration).

Classroom Experiments

Bonnie and Lada explored the problem in their own grade 8 classrooms. Each lesson was videotaped and analyzed for evidence that the teachers were attending to the social semiotic challenges of the problem while they orchestrated a whole-class conversation. In addition, Bonnie explored the problem with new students a second time in year two of the project, and was again videotaped. In the presentation of this paper, I will discuss these findings.

Concluding Comments

The teachers in our lesson study group are exploring the social semiotic challenges of teaching and learning mathematics. The Fishpond problem was an excellent non-routine problem for them to experiment with, as it allowed them to attend more rigorously to the linguistic and diagrammatic facets of problem solving (Radford, 2004). Since speech, inscription, diagram and gesture together form "semiotic bundles" (Arzarillo & Paola, 2007) by which participants construct a multitude of meanings, the process of apprenticing (and submitting) to semiotic norms is complex and multi-faceted. As seen in the cases of Lada and Bonnie, the teachers tried to access and leverage different semiotic tools to facilitate student engagement with the problem, without explicitly telling their students that the square could be tilted. The "reality" of the problem proved to be significantly distracting in the more linguistically diverse classroom, and the framing of the problem in terms of transformations seemed to facilitate in priming the students in Lada's class to think more expansively about the invariance of shapes. Bonnie's second experiment reveals that she has increased her skills at framing the non-routine problem and at negotiating the complex conjunction of the mathematics register and everyday language.

The social semiotics approach allowed the teachers to focus on some of the socio-cultural habits of mathematics problem solving. In particular, they studied how the iconic and the symbolic are fused in student diagramming and how this points to the complex coupling of everyday and esoteric sign systems. They studied strategies for circumventing the student resistance to the artificiality of word problems, and discussed how words like "imagine" might be taken up differently by different students, in some cases as a command to "make believe" and in others as a command to "make an image", and that this difference spoke to their students' cultural assumptions about the artificiality of word problems. They also learned to decode alternative word choices in terms of ambiguity (on, at, in) and studied how to move back and forth between material verbs (sitting, cutting, tilting, folding) and the relational verbs (to be and to have) which predominate in formal written mathematics.

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