The 'Ganak' Exploration: Helping Children to Learn About Numbers

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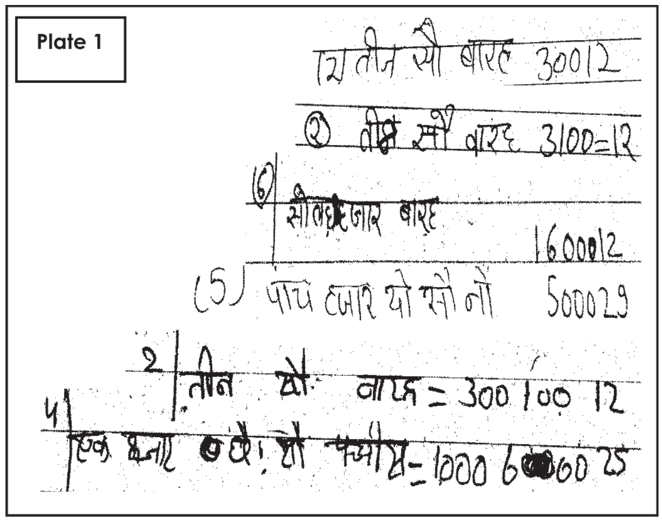
This paper explores the Ganak –a stick abacus having thin rods fixed vertically that can fit in exactly 9 beads; for the tenth beat one has to remove all the beads and put one bead in the next rod. The abacus is constructed to help children learn more about the number system and the research question framed was– how can I use the idea of the Ganak to teach more about the numerical representation system to grade 6 children in activity based classes? The intervention was done in a village school, with grade six children who have been using numbers. A base 3 game, based on Ganak was developed. It was observed that the game gradually conveyed properties of positional notation system without explicitly taking children into the complexity of different bases. The children enjoyed playing it, gradually understood the rules and developed their own games for different bases. Games developed by the children indicated that they could grasp the concept, used in the abacus. Given the grasp of the abacus principle, children could figure out the relation between the face value of digits and the place value of the same for given numbers. The Ganak helped the children in understanding the meaning of "+" and "***x***" signs in expanded form of numbers.

Keywords: Abacus, Positional notation system, Ganak

Introduction: The Starting Trigger

At times children get confused in mathematical concepts due to its 'complexity' and differently sense these, which we (adults) call 'mistakes'. However, these mistakes can lead us to children's level of understanding and remedial work required, to 'correct' their understanding of a given concept. During one of my interactions with grade 6 children of Kulamari village, I found that they were 'confused' about the standard numerical representation of the numbers, based on the Place Value System (PVS). I found in the empirical data of the grade 6 (Plate 1) that children are confused in the use of zeros while re-presenting a number. For example: one thousand three hundred twenty four was written as 100030024, which seems sensible. Their difficulty lies with the basic characteristics of PVS. Research shows that at times children find it difficult to differentiate between the face value of each symbol in a number and the complete value of the same symbol (Varelas & Becker, 1997).

Plate 1



Concept of positional notation system or PVS

Ross (2002) enlists the following characteristics of PVS or Positional Notation System *"Our numeration system is characterized by the following four mathematical properties;*

Additive property – The quantity represented by the whole numeral is the sum of the values represented by the individual digits.

Positional property – The quantities represented by the individual digits are determined by the positions that they hold in the whole numeral.

Base-ten property – The values of the positions increase in powers of ten from right to left.

***Multiplicative property** – The value of an individual digit is found by multiplying the face value of the digit by the value assigned to its position, example $1324 = 1 \times 10^3 + 3 \times 10^2 + 2 \times 10^1 + 4 \times 10^0$.*

What do I mean when I say PVS? We do not need a PVS so long as we use a number verbally but we require PVS when we start writing numbers. We use ten symbols (1, 2, 3, 4, 5, 6, 7, 8, 9 and 0) in the PV based decimal system. Each possible distinct combination of these symbols represents and is associated with a unique number. The PVS is a system for generating all these combinations to get a unique sequence of counting numbers. This also known as Positional Notation System

Other representations of the decimal system

The early Egyptian number system had symbols for 1 to 10 and for all multiple of 10 (www.geocities.com). Using these symbols, one thousand two hundred thirty four, will be written as shown in Plate 3. The Egyptian system differs from PVS in two important aspects: We need to keep generating infinitely new symbols for different powers of ten.

Plate 2

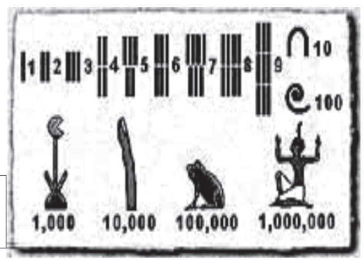
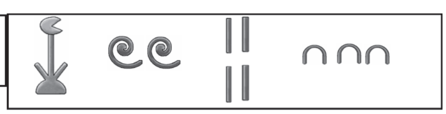


Plate 3

The above system does not require specific position for each of the symbols. If I write 1234 using Egyptian symbols, I may have multiple ways of writing such as plate 4 & 5. Both these combinations stand for the same number because the individual symbols stand for the value not due to position but due to shape in the number and 'Zero' is not needed here. On the other hand, the PV based decimal system uses only ten symbols including zero. These symbols acquire different value when placed at different position in a number. Any change in the order of the symbol will represent a different number.

Plate 4

Plate 5



Importance of zero as place holder

Historically, this positional nature of the representation led to discovery of zero – '0'. Let us try to represent a number in PVS without using '0' to understand its significance. Suppose we have to represent one thousand thirty two. We can write 1 32 leaving a blank/gap in place of zero at right to left third position (hundred's place). But it may create confusion among numbers like one hundred thirty two (132) or ten thousand thirty two (1 32) because of mistake in judging the gap. So the zero is introduced to hold the 'empty position' and now we write 1032 for one thousand thirty two.

How numbers are taught?

There is an unofficial debate among mathematics educators on the method of teaching numbers. Realistic Mathematics Education (RME) group argues in favor of delaying place value and focusing on the development of number sense at primary grades. I will further call it **whole number approach (WNA)**. The other approach which suggest that PV be taught as a basis to understand number. Various materials have developed for the same. I will call it **Place value based approach (PVA)**. Literature and material survey throws light on these two approaches as follows.

Whole number approach (WNA)

The following paragraph draws upon works of Menon (2004), Kamii and Joseph (1988). WNA advocate that counting on the ten-structured color-coded beads string and then jumps on the empty number line keep the numbers whole and not get into the place value i.e. in 32, 3 tens and 2 ones (Menon, 2004). Some of the WNA materials are enlisted as; colour-coded 100 beads string – Ganit mala (Heuvel-Panhuizen, Buys, & Treffers, 2001; Menon, 2004); Empty Number line (Menon, 2004), Strategies of mental arithmetic (Heuvel-Panhuizen, Buys, & Treffers, 2001; Kamii & Joseph, 1988)

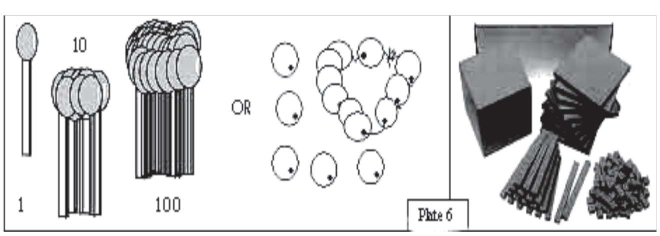
Place value based approach (PVA)

We have been traditionally teaching the concept of place value at primary grades (IGNOU, 2001a, 2001b). Children are expected to learn to write the number in columns of hundreds, tens and units etc. The teaching of place value is advocated so that algorithms (which are prevalent in our textbooks since colonial times) of addition, subtraction, etc. are acquired by children. These algorithms are believed to be the most efficient ways of solving arithmetic. Also it would be difficult for a child to understand the big numbers without the understanding of place value. Materials have been developed to communicate the concept at primary level. To enlist a few: the matchsticks bundle or Beads mala *Khushi Khushi Class 2*, (Eklavya, 2002), *the Dienes block* (Zoltan, 1950), *Khushi Khushi Class 4, pebble card, Khushi Khushi Class 3, the "snap, clap, tap" game, HBCSE Mathematics Textbook of Class 3* (HBCSE, 2001). The WNA methods and materials are for use to begin the teaching of numbers at elementary level and it is suggested that children be encouraged to count and acquire the sense of combinations of digits so as to catch the logic that the numbers are arranged in the group of 10. Gradually the children would acquire the meaning, 32 means 30+2.

At higher primary grades (grade 4 and 5) when a child's cognitive level is comparatively higher, I would suggest using the materials recommended by the PVA as well. These materials start with the grouping logic which a child had experienced during the WNA. In spite of strengths, both the approaches do not logically lead to positional notation system.

The material review (used in PVA)

There are types of PVA materials available which are conceptually similar (Plate 6). To show 45 using these materials, we will take 4 bundles of ten matchsticks and 5 single matchsticks. All other PVA materials are more or less on same principle. These materials give meaningful experience of numbers, but PVS does not logically arise through these materials. It is imposed as an external account keeping system with the unit's/ten's columns in certain sequence to make similar with the PVS. Each new bundle is different in value from unit matchsticks. For example the two matchsticks stand for 2 and two bundles of ten matchsticks stand for 20 but not because of the position but it actually has 20 matchsticks. These materials can best fit the early Egyptian number system and hence have the same difficulties as we saw in early Egyptian number system. Dienes wrote *"A set of multibase blocks, which I introduced in England, Italy and Hungary in the 1950's. Educators today use the "multibase blocks", but most of them only use the base ten, yet they call the set "multibase". These educators miss the point of the material entirely".* These materials are no doubt good for teaching grouping and/or base 10. Using an external column system of unit, ten and hundred we can give a child a very concrete example of face value and place value of any number. But strength and need of the additive (multiplicative) composition of numbers is simply lost. This means, the Positional and Multiplicative properties do not exist as a need in this system. This calls for some additional tool or tools to strengthen these properties of number.



The Ganak as a solution

An attempt was made to use the open abacus (known as Ganak) as a tool to teach decimal and place value, mentioned in *Bal Vaigyanik*, HSTP text book of Grade 6 (Eklavya, 2000). It has thin rods fixed vertically, can hold exactly 9 beads. We begin by putting beads in the right-most rod and continue doing it till 9 beads. When there is no space left for the next beads, we take out all the beads and place one bead in second rod from right. This bead is equal to ten beads. Here the bead stands for the value due to position. This situation is similar to the numerical representation of 10. The unit's place in the Ganak is empty indicating 0 and the ten's place has 1 bead standing for ten units or one ten. This is closer to our number system, unlike other tools of PVA. Thus Ganak seems conceptually appropriate to convey the positional nature of our number system. But even in Ganak, naming of the rods, in certain way, is arbitrary. If we take this arbitrary position then the Ganak can help children to understand the strength and need additive (multiplicative) composition of numbers.

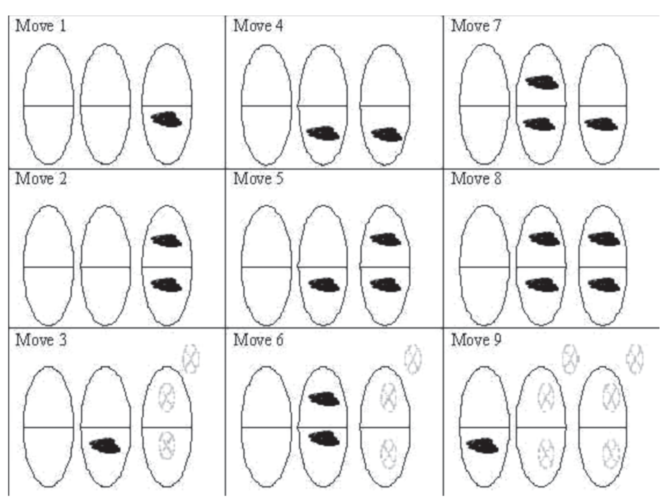
Plan of Intervention

Objective

The above review gives a research context that there is a need to introduce some additional tools and method to help children learn the positional nature of numbers. Working out some teaching method of place-value system or the efficacy of Ganak weren't the main objectives. My study is mainly about the 'exploration' of the Ganak. I planned to begin with different base the classes. Zoltan Dienes (2001) also said "Different bases be used at the start, and to facilitate understanding of what is going on, physical materials;' embodying the powers of various bases should be made available to children. (Dienes, 2001)

The base-3 game

To start with, I designed a game, exactly based on the Ganak. The game was to give exposure of the positional property of numbers in base-3 without explicitly talking about the bases. I would call the game as base-three game in this paper, is shown below.



Third, sixth and ninth moves are crucial because the children have to consolidate the moves and shift the pebble to the next column.

Children were made to play this game many times. They were asked to represent it on paper initially with the pictorial symbols and gradually shift to the numerical symbols. I helped them naming the boxes and use it to calculate the moves for the pebble combination.

As the next step, I encouraged all of them to design their own games just by extending the parts in the boxes, based on their experience of base-three game. Finally I brought the abacus in the class and made them see exactly same rules in base-10 system.

The study method

I selected grade 6 of a village middle school, Kulamari, Hoshangabad, which has 22 students (boys and girls) to conduct this research. Before the Ganak intervention, I spent almost a month with the children using the existing materials. Children played various counting games on the Ganit Mala with it and did some amount of addition and subtraction etc. Then I made the children play the exchange game etc. using the pebble card. As I expect that the Ganak be used directly at middle school level, by then the children have had exposure to these materials. But in this school the children did not use these materials. I took 12 days of intensive classes of one clock-hour per day. I collected three types of data; audio records of the intervention classes – *(Transcriptions – Tr),* writing of the memos and observations on regular basis – (*Classroom Observations – CO),c*ollection of children's actual class-work like answer sheets etc.- *(Work Sheets – Ws).* I analyzed these data through different analytical questions. Students' actual work could tell me how the children progressed. My classroom observations and memo could tell me about the on-field content and sequence and the audio transcription of the classes could tell me the teaching pedagogy and the class responses.

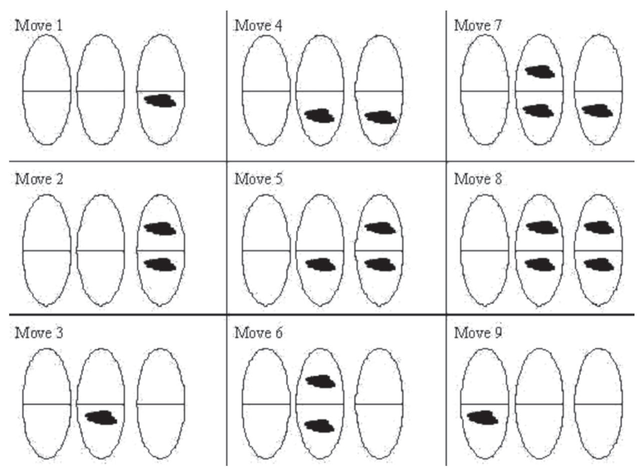
Rationale of the Study

I decided to conduct my research at Kulamari, Hoshangabad district, located in Madhya Pradesh, India. The village Kulamari is 8.5 km from Hoshangabad. In this research work, the place is not a matter of much concern. Since Eklavya's team is located at Hoshangabad city, it was convenient place for me to arrange things and try out materials and methods. Also I had been visiting the Govt Middle School, Kulamari since last few months and had developed a good relationship with the sixth graders. Since the conceptual context of my research suggests that the children should have exposure to the positional notation system to elaborate their understanding of the concept of place value and the concept of positional notation system requires little higher cognitive level. So I decided to work with grade 6 students.

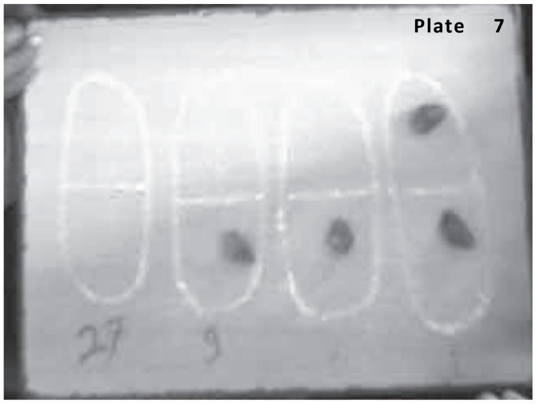
Originally Ganak was used in grade 6 during one of the Eklavya's Educational projects (HSTP). The HSTP textbook "Bal Vaigyanik" also justifies conducting the study in grade 6. Bal Vaigyanik, after its intensive field-based reviews, has placed the chapter on positional notion as it is now. So I assumed that sixth graders were comfortable using the Ganak. *"The need for a remedial course, in decimals had been established. The resource group members put their heads together. A trial draft was ready by the 1974 orientation course. In this novel approach, the children were expected to make an abacus of their own and use it to understand the significance of place value and hence decimals. Two research students took up the challenge of designing an appropriate abacus. What came out of this effort has now come to be known as the GANAK – it is an assembly of six straight wires fixed vertically and equidistant on a wooden platform"* (India today Dec, 1977).

Results

Study intervention and responses

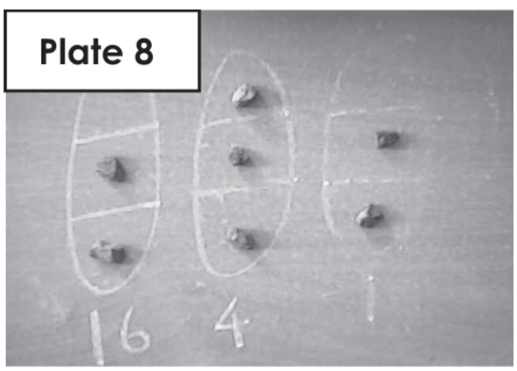


For the children, the base-3 game was a game of two parts. (*Do Khand wala khel*). I had to make some changes in the designed base-3 game, I removed the complexity of third, sixth and ninth moves after the day 1 class. I removed step of keeping the third pebble outside the first box and the then replacing all three by one pebble in next column. Then the new game was – At 3rd, 6th and 9th moves, they have to consolidate the moves



and replace it by a pebble to the next box or column but now they were asked to simply put third pebble in next column. {CO: Day 1, 29 Jan 2009; Day 2, 5 Feb. 2009}

I made the children play this game enough number of times so that they could get hold on to it at individual level. They have extended it means 4, 5 boxes (Plate7). Then I made them represent it on paper with the pictorial symbols. They learnt to play and read first and then write. Now they were able associate each single pebble combination with a unique number in ascending order. {CO: Day 2, 5 Feb.; Day 3, 6 Feb.; Day 4, 10 Feb. 2009} Next I concentrated on naming the boxes (position) and its relation with the moves (numbers) as shown in Plate 7. {CO: Day 5, 11 Feb. 2009}



I attempted to do some +/- algorithm using pictorial representations. Only 2 out of 18 could do it. So I dropped it. {CO: Day 6, 12 Feb.; Day 7, 13 Feb}.

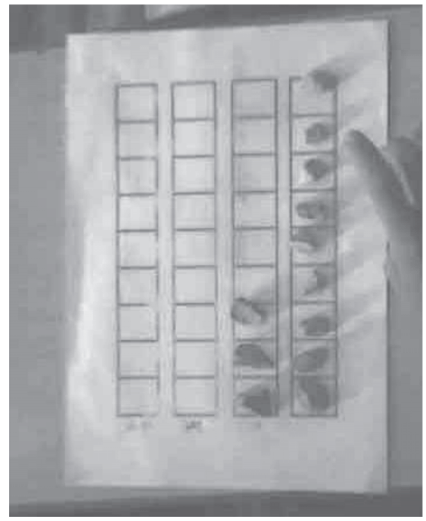
Then I made all of them to work in groups and design their own game just by extending the parts in the boxes which means more than base three. All three groups did it successfully (Plate 8).

In few of the last classes we reached a stage where children were converting various base numbers into the base ten numbers. (I did not use these terms base-ten, base-three etc)

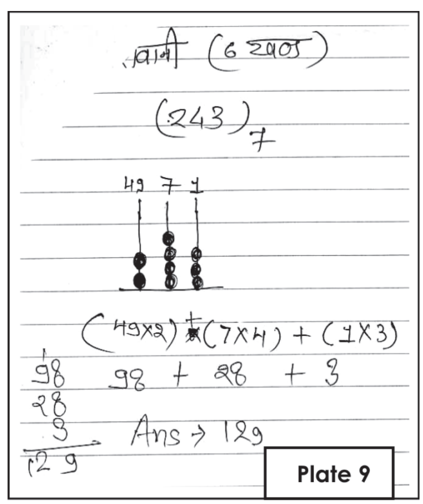
"It was a marvelous for me that two of them wrote the number in the form of additive (multiplicative) equation using base ten numbers in the game designed by them" {CO: Day 7, 13 Feb.; Day 8, 14 Feb. 2009}

Two children did the *additive (multiplicative) equation* (Plate 9)

I thought to stop here and conclude the activity and I brought the stick abacus for doing all with base-10 (sequence, naming the place, picture representation and then numerical representation). But it was not satisfactory class. {CO: Day 9, 16 Feb. 2009}



I had re-planned the last classroom activity and planned to repeat all activities with the base-3 game at the individual level. {CO: Day 10, 19 Feb. 2009}. The day7 experience led the basis for me to work with children further and to allow them to attempt to write the number in the form of additive (multiplicative) equation. Many of them (10 out of 12) could write the additive (multiplicative) equation for their game.



So at the end of the intervention I made them play with 10-base game (game form of the stick abacus) in the same manner as they had been playing the base-3 game. {CO: Day 11, 20 Feb.; Day 12, 21 Feb. 2009}. I ended the intervention here as I felt that I had successfully explored the abacus activities to help children understand the positional notation system.

How children learn/progress (a case study)

This section tries to understand how an individual child progressed. 10-12 children were present almost throughout the intervention classes. I selected 10 students out of them and prepared narratives of each using their actual work to understand their performance in the intervention classes. I would take a student A as a case study but also talk about other and the class in general.

A's Back-ground

General assessment - A is an average child, is recognized as 'disturbing element' in the class but I found him attentive though at times little distracted. 'A' had been absent for 2-3 days during the course of the intervention classes in the initial period. But there were lots of back and forth in the activity design so I hope he did not miss much.

Pre-assessment – I had conducted pre-intervention test just before the Ganak exploration classes. I were questions to see children's number sense for which I also conducted classes using the Ganitmala etc and to see whether they could find pattern in other base numbers.

Question-wise answers {**A's Ws: pre-test; answers of questions 1 to 4**}

It was asked to show a number in picture of Ganitmala- Almost all did it correct including 'A'. Question about the empty number line up to 500 (a straight line on which the first and last numbers are marked at both the ends and children are expected to divide the line according to the least count s/he has chosen). At the ganitmala activities 'A' had been performing fairly well in the class. In the pretest, his answer apart from the judgment of distance was correct.

The question was to calculate how many bundles of single, 10, 100 matchsticks are possible to make, in the number 352 - In the answer sheet he made some calculation mistakes. He wrote - *"34 bundles of ten matchsticks each, Three bundles of 100 matchsticks each, one bundle of 52 matchsticks"*.

0	22	110	132	220
1	23	**111**	133	221
2	30	112	**134**	222
3	31	113	201	**223**
10	**32**	120	202	230
11	33	121	203	231
12	100	122	210	232
13	**101**	**123**	**211**	233
20	102	130	212	**234**
21	103	131	213	**300**

The interesting part is that he did not do it mechanically but he tried calculating, though ended up with incorrect answer. I gave a chart of the base 4 numbers (using Indo-Arabic symbols) up to 301 (forty nine) with few blanks left at various places. A's answer is written in bold in the adjoining chart. I don't know whether 'A' found logic/pattern in the given number. He wrote 13 after 12, 101 after 100 but he also wrote 234 after 233. Certainly 'A'did not see these numbers have only four symbols (0, 1, 2, and 3).

Pre assessment of the class – I had given a pre-test but still I felt that I would be considering all my classroom experiences and other tests, which I would be doing at various stages till the end to assess children's understanding.

Classroom participation and performance

I started with two beads abacus (the base-3 game). The method of moving is same as we do in the 9 beads Ganak (base ten abacus). In this three-base abacus, the single bead in the second column values 3. The value of beads in any column is 3 times the value of beads in the previous column. We had drawn this abacus on floor and instead of the beads we had used pebbles. A's performance in the class was average as I had student like Sh and Pre who participated and performed better than A and I had students who did not reach up to the level of A. I mean by the term "perform better" is they had grasped things very fast and also helped me in conducting class and helped peers.

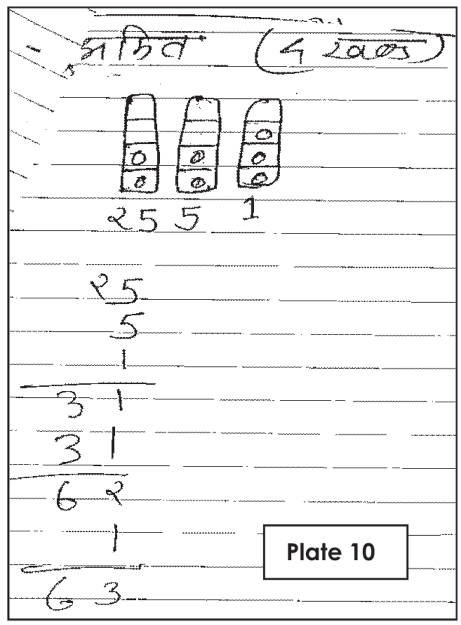
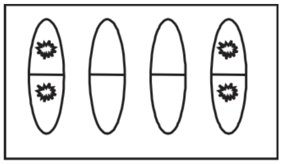


Plate 10

A's performance and participation – 'A' had learnt the base 3 game and found the logic of the game but at times A made miscalculations in the pebble positioning and was poor at representation part. In the middle test it was clear that for small numbers he made no mistakes but with the big numbers he did. But identifying moves, for the pebble positions 'A' was correct.

When he was asked to identify the number given in the adjoining diagram he identified correctly. Also when he was asked to show less than 4 and more than 4, he had drawn 3 and 5 correctly. 'A' was shown a picture depicting 28 in the base 3



game and asked to show count on numbers up till 34 he made mistakes. {A's Ws, mid-test}

Later at the time of writing of base-3 number numerically, A wrote most of the numbers correct. He had been making mistakes but he starts playing it and correcting himself. {A's Ws, sheet 3}

I asked the children to design and play any other-base games in groups. All the groups did that. After that I gave A, a combination of pebble in base-five game and asked him to tell me the number of moves required for reaching at this stage. He took a little help from me and calculated. {A's Ws, sheet 4}

I am giving herewith the scanned image of his work. He worked beyond my imagination. First of all he named the boxes 1, 5 and 25. Then he added first 25+5+1 for the first row of pebble then doubled it and then added the unit box's left out pebble and got the answer 63. (Plate 10)

Towards the last phase of my intervention, I made all of them to convert various base numbers into base ten. A also did few of them although there were few errors in the calculations. When he was asked to tell how many moves were required to get (1221)pebble combination in the game that has 5 parts of a box, his answer was the adjoining scanned image (Plate 11) He did few more similar questions. 'A' wrote the full equation of conversion of 6-base and 7-base into decimal numbers (Plate 12). He named the places (6 to the power 1, 2, 3 etc and 7 to the power 1, 2, 3, etc). He uses the addition and multiplicative sign and also understands the meaning of the answer that it is the number of moves.

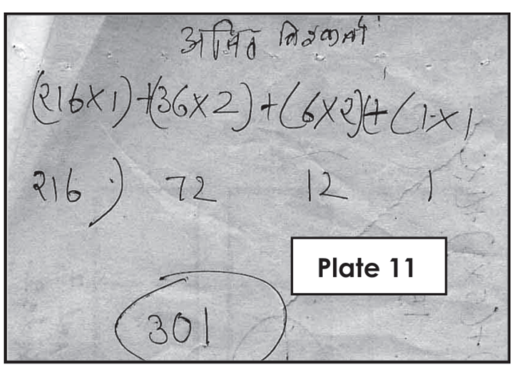


Plate 11

In the end I made all the children play the same game in base 10 (or the stick abacus on ground which can fit in exactly 9 pebbles) they were finding it very exciting that the decimal numbers were 'popping out' through the game which they had been playing for last few days.

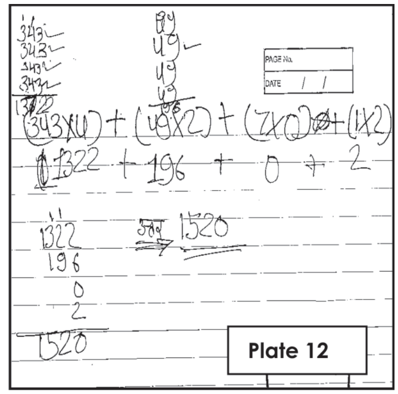


Plate 12

A's works suggest that he has got a sense of additive (multiplicative) concept of positional notation system in other than base-10 numbers and this experience will help her to understand the same concept in base ten as I found all selected students' performances were more or less similar as A's, during the intervention and the all intervention assessments. Some or the other interesting responses were found in many other narratives and one of them, the interesting horizontal addition, is observed in A's narrative summary.

Conclusion and Recommendations

It is also worth sharing that I took up this small project as an attempt to learn the research methodology so this should not be seen as 'cutting edge' research but certainly it can be treated as a pilot study. Although I am interested to see how the Ganak can be explored. I am still left with few issues/questions on the present study to be understood as a pilot study for the efficacy of Ganak to teach place value.

Conceptual Issues: The connection between the positional notation system and our numbers seem a jump. I only assume that this understanding will help a child to understand number better in future! Why place value should be taught?

Issues on method: I have not been able to carve out enough justification for the choice of grade 6. The teacher's style also plays an important role, but the study could not incorporate it.

Apart from this, the study surely tells that the **Ganak is a powerful tool**. If it is felt that **we should reinforce the place value system of numbers in middle schools**. Then this trial can be treated as an attempt to recommend Ganak to educational agencies.

Acknowledgement

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