Set Theory Made Easy Through Set Theory Multi Operation Machine

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The set theory has gained importance in modern mathematics. The concept of set theory being recent in mathematical world, great care has been taken to explain each detail in such lucid style for high school students; a mathematical mind should have least difficulty in grasping the basic concept to the subject. In order to make the content of the subject interesting and convincing to students always new teaching aids are in demand (or required), I have prepared a audio visual aid – set theory multioperation machine. Through this apparatus students can easily study and understand quickly about the set theory definition and its operations, Venn-diagrams, theorems, properties and daily life situations. The importance of this machine is to develop concentration and interest to study mathematics without black board and chalk. I think this model would be very popular for mathematics students.

Keywords: Set theory, Audio-visual aid, Teaching aid

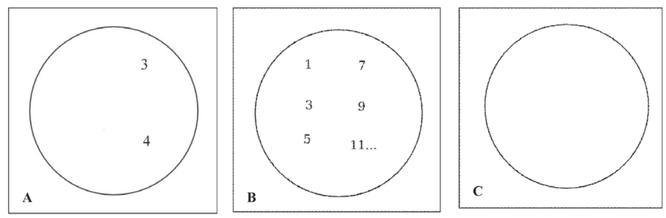
Introduction

The concept of set theory being recent in mathematical world (Rich & Schmidt, 2009; Rudin, 2009), great care has been taken to explain each detail in lucid form for high school students, a mathematical mind, should have least difficulty in grasping the basic concept to the subject. In this regard, I have developed a model 'audio-visual aid' *Set theory multioperation machine*. I propose that through this apparatus students can easily study and quickly understand the set theory, definition and its operations, Venn-diagrams, theorems, properties and daily life typical problems.

George Cantor put forward the concept of a set "*A set is a collection of well defined and distinguish objects*". A set is represented by two ways:

(i) Set builder form i.e. A= {x: x is months name of the year started from J},

(ii) Tabular form i.e. A= {January, June, July}



(i) Finite Set (ii) Infinite Set (iii) Empty Set

Operations on sets can also be represented by Venn-diagram. It is a pictorial representation of operations on sets in which all sets are represented by circles and elements contained with specific manner within it. The number of elements may be finite, infinite or empty.

Model – Working Procedure

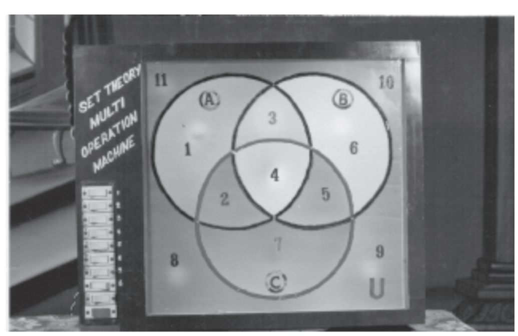
In a box, there are eleven bulbs which are representing elements of different colours, connected with an electric circuit through related individual switches affix on left corner of screen. Different regions of set on screen can be easily shown by different circles or parts of circles by glowing. There are only four operations on sets like:

(i) Union of sets

(ii) Intersection of sets

(iii) Difference of two sets

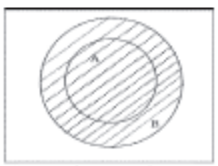
(iv) Universal set and complimentary set



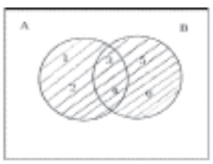
There are related laws on set operation that can be easily understood by this pictorial representation. We use this apparatus that can visualize the set theory and its operations. In this process set operations can be studied by teachers and learners in a very short time. Students can easily study all the Venn-diagrams and note them topic wise.

Union of Two Sets

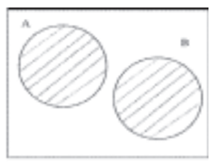
Activity work: First, we should show different examples on screen about union of two sets and then reach to definitions of two sets:



(i) $A \cup B$, when $A \subset B$



(ii) Let $A = \{1, 2, 3, 4\}$ $B = \{3, 4, 5, 6\}$ $A \cup B$ when neither $A \subset B$ and $B \subset A$

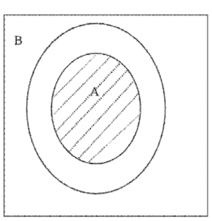


(iii) $A \cup B$ $A \cup B$ when A&B disjoints sets

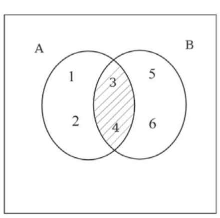
Definition: The union of two sets A and B is the set of all those elements which are either in A or in B or in both. It is denoted by $A \cup B = \{1, 2, 3, 4, 5, 6\}$ $A \cup B = \{ x : x \in A \text{ or } x \in B\}$ can be represented by glowing parts on screen. This operation can be shown by 6 bulbs and hence this operation can be practiced by each student himself like for $(B \cup C)$, $(C \cup A)$, $(A \cup B \cup C)$ etc.

Intersection of Two Sets

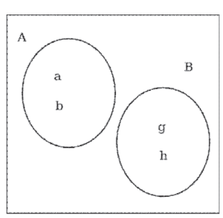
Activity work: Different cases can be shown on screen and then conclude the related definition.



(i) $A \cap B = A$ When $A \subset B$



(ii) $A \cap B = \{3, 4\}$ When neither $A \subset B$ nor $B \subset A$

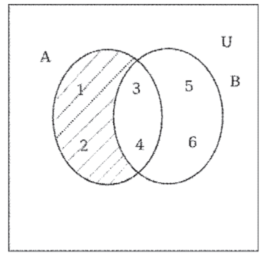


(iii) $A \cap B \neq 0$ When A & B are disjoint

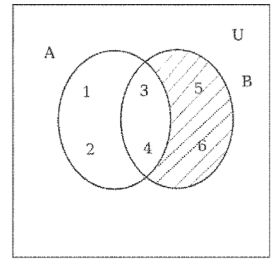
Definition: The intersection of two sets A and B is the set of all those elements which can be represented by glowing parts on screen. For practice, this operation can be practiced by each student on machine taking other examples $(B \cap C)$, $(A \cap C)$, $(A \cap B \cap C)$, etc.

Difference of Two Sets

First we show some pictorial (activity) examples on screen such as:



(i) We switch on for set A, then switch off the Bulbs 3 & 4, which elements belong to B
We get A -B = {1, 2}



(ii) We switch on for set B, then switch off the Bulbs 3 & 4, which elements belong to A
We get B-A = {5, 6}

Definition:- The difference of two sets A and B written as A-B, is the set of all those elements of A which do not belongs to B.

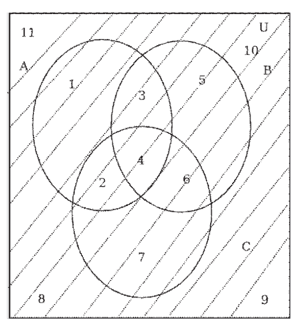
Thus, $A-B = \{x : x \in A \text{ and } x \notin B\}$

Similarly, $B-A = \{x : x \in B \text{ and } x \notin A\}$

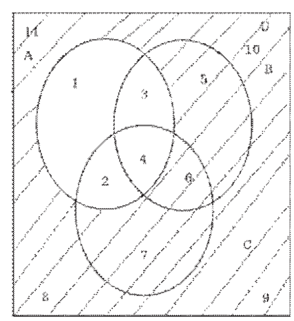
Remarks: It is clear that A-B ‘“B-A i.e. it does not hold commutative law. Hence (A-B) can be obtained by leaving the elements of B from Elements of A. This activity can be practiced by students for (A-C), (B-C), $A-(A \cap B)$, etc.

Compliment of a Set

We will shown Venn-diagrams U for Universal set and A, B, C are its subsets in it. Generally universal set can be represented by rectangle as on screen of this audio-visual kit.



(i) U= Universal Set



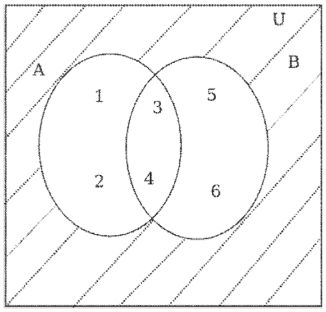
(ii) (U-A) = A or C_A, Difference Set of A

Definition: Let U be the universal set and A is any subset of U, then compliment in A, is denoted by A' or $A^C = (U-A)$.

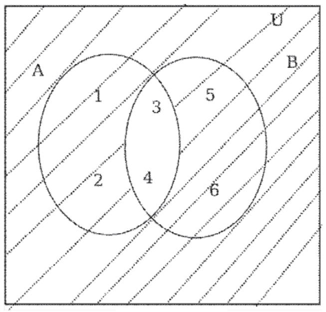
Thus, $A' = \{x: x \in U \text{ and } x \in A\}$ i.e. $A' = \{5, 6, 7, 8, 9, 10, 11\}$

Similarly, $B' = \{x: x \in U \text{ and } x \in B\}$

Note: This activity can be practiced by students for C′, (A∪B)′, (A∩B)′ (A∪B∪C)′, etc. Some other results can be easily proved by this model — U′ = ϕ (ii) (A′)′ = A, (iii) A∪A′ = U, (iv) A ∩ A′ = Φ etc.



1. (A∪B)′



2. (A∩B)′

Applications of Model

With the help of these four operations, we can prove other laws on sets and its application in daily life problems.

Laws of set operations

i. Idempotent law :- (a) A∩A = A (b) A∪A = A

ii. Identity law :- (a) A∪Φ = A (b) A∩Φ = Φ

iii. Commutative law: - (a) A∪B = B∪A (b) A∩B = B∩A

iv. Associative law: - (a) A∪ (B∪C) = (A∪B)∪C (b) A∩ (B∩C) ∩C = (A∩B)

v. Distributive law: - (a) A∪(B∩C) = (A∪B)∩(A∪C) (b) A∩(B∪C) = (A∩B)∪ (A∩C)

vi. Demorgan's Law: - (a) (A∪B)′ = A′∩B′ (b) (A∩B)′ = A′∪B′

vii. Symmetric difference of two sets: - A Δ B = (A-B) ∪ (B-A)

Some important theorems on number of elements of sets based on problems in daily life.

If A, B, C are finite sets and U be the finite universal set then, like n (A) = number of elements in A

(i). n(A∪B) = n(A) + n(B) – n(A∩B)

(ii). n(A-B) = n(A) – n(A∩B)

(iii). n(AΔB) = n(A) + n(B) – 2n(A∩B)

(iv). n(A′∪B′) = n(A∩B)′ = n(U) – n(A∩B)

(v). n(A′∩B′) = n(A∪B)′ = n(U) – n(A∪B)

(vi). No. of elements belonging at least one of the three sets

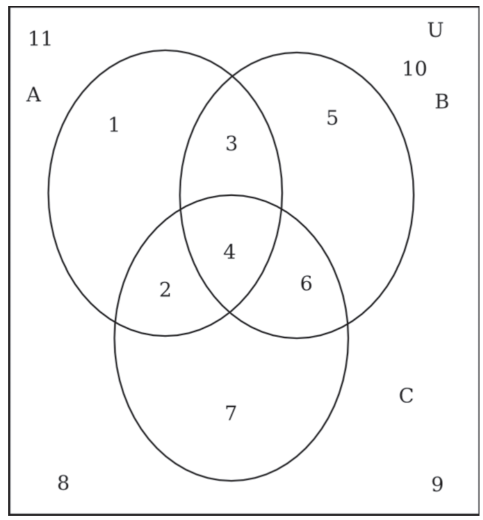
n(A∪B∪C) = n(A) + n(B) + n(C) – n(A∩B) – n(B∩C) – n(A∩C) + n(A∩B∩C)

(vii). No. of elements belonging in exactly two of the sets

A, B, C = n(A∩B) + n(B∩C) + n(C∩A) – 3 n(A∩B∩C)

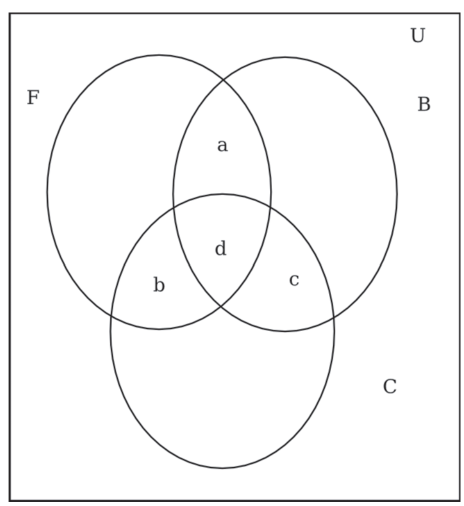
(viii). No. of elements belonging exactly one of the sets

A, B, C = n(A) + n(B) + n(C) -2n(A∩B) – 2n(B∩C) - 2n(A∩C) + 3n(A∩B∩C)



Problems based on daily life can be solved through this kit.

Example: A college awarded 38 medals in Football, 15 in Basketball and 20 in Cricket. If these medals went to a total of 58 men and only three men got medals in all the three sports. How many received medals in exactly two of the three sports?



Solution: Consider on the Venn-diagram

Let a = No. of men who got medals in Football & Basketball only

Let b = No. of men who got medals in Football & Cricket only

Let c = No. of men who got medals in Basketball & Cricket only

Let d = No. of men who got medals in all three sports F, B, & C

Given $n(F \cup B \cup C)=58$, $n(F)= 38$, $n(B) = 15$, $n(C) =20$, $n(F \cap B \cap C) = d = 3$

Now, $n(F \cup B \cup C) = n(F) + n(B) + n(C) – n(F \cap B) – n(B \cap C) – n(A \cap C) + n(F \cap B \cap C)$

Therefore, $58 = 38 +15 +20 – n(F \cap B) –n(B \cap C) - n(F \cap C) + 3$

⇨ $n(F \cap B) + n(B \cap C) + n(F \cap C) = 76 - 58 = 18$

⇨ $(a + d) + (c +d) + (b + d) = 18$

⇨ $a + b + c = 18 – 3d = 18 – 3 \times 3 = 9$

⇨ No of people who got medal in exactly two of the three sports = 9

Advantages of This Model

In order to make the content of the subject interesting and of convenience to students, always new teaching aids are required and in demand. So the importance of the *"Audio Visual Aid — Set theory multi operation machine"* is to develop concentration and interest to study mathematics without black board and chalk. I propose this model would be very popular to learn mathematics for students effectively.

References

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Rudin, W. (2009). *Algebra.* NewDelhi: Prentice-Hall of India.