Finding Teaching-Learning Paths in The Domain of Multiplicative Thinking

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The idea of hypothetical learning trajectories in primary mathematics has re-emerged as a useful way of organizing and disseminating complex research findings about student learning in specific topic domains. Learning trajectories are frequently described as conjectured progressions of learning experiences that students encounter as they move from informal to complex, refined and powerful ideas over time. Learning trajectories in specific areas have been viewed as bridges that connect "grand theories" in education with specific theories and instructional practice (Sarama & Clements, 2009).

While this idea has been productive in the topic domain of whole numbers, addition and subtraction, it has been difficult to extend it to the topic domain of multiplicative thinking. One reason is that this domain has an extensive vertical elaboration including topics at almost every level of education. Thus, it includes whole number multiplication and division in the primary grades, fractions, ratio and proportion in the middle grades, and linear functions in the secondary grades and beyond (Vergnaud, 1988). Several researchers identify multiplicative thinking as the most important core topic area in the middle school, since the most critical elaborations and development of informal ideas takes place in these grades centred around the topics of fractions, ratio and proportion (Carpenter, Fennema, & Romberg, 1993). Much research on the domain of multiplicative thinking has focused on the middle grades and on the topic of fractions which is a foundational concept for the domain.

One may identify three broad sets of research findings as critical to the process of identifying hypothetical learning trajectories. The first is knowledge about children's action schemes and their elaboration in the course of interaction with everyday situations, and in didactical contexts that may include a variety of problem situations and concrete embodiments. With regard to multiplicative thinking, this strand of research, greatly influenced by Piaget, is the most developed and has led to an understanding of a variety of schemes that children develop such as equipartitioning, unit iteration, unitizing and one-many correspondence. A second set of research findings that is needed is how children bridge action schemes and symbolic routines. As children learn to solve more complex problems they must increasingly rely on the mathematical power made available through symbolization. Yet, they must make sense of the symbols and their transformations, drawing on their knowledge of schemes, of situations and on previous symbolic knowledge. In the domain of multiplicative reasoning, the primary symbolic tools consist of the fraction notation and the arithmetic of fractions. These symbolic tools consolidate and extend the ability to represent and manipulate multiplicative relations. They provide the tools to deal with the full range of situations involving proportionality and also prepare the student for algebra. This strand of research in the topic domain of multiplicative reasoning, which has fruitful connections with the research in learning algebra, is relatively less developed.

A third strand of research that contributes to identifying learning trajectories seeks the sources and support for learning in the culture and illuminating the relation between out-of-school mathematics and school mathematics. Many everyday situations in which people deal with quantities involve proportional relations and call for multiplicative thinking. Students, especially those participating in household income generation are likely to be familiar with such contexts. Given the ubiquity of proportional relations, and therefore the importance of multiplicative thinking, such studies can contribute to both identifying general principles and developing localized versions of learning trajectories. Studies that uncover out-of-school knowledge and look for opportunities to connect it with school mathematics may confront complex issues because the culture that students are a part of is varied depending on location and social stratum and also changing.

Such studies also relevant to broader issues of equity, the relation of education to society, to social change and to empowering individuals.

Some theories specific to the domain of multiplicative thinking have a broader scope and provide a basis for integrating the findings of the three strands of research described above. One example is the classification of proportion problems by Vergnaud (1988). Another example, the sub-construct theory of fractions (Kieren, 1993), is an analysis of the different types of situations in which the arithmetic of fractions can be applied. It unifies the interpretation of the fraction symbol across these situations into a small number of different sub-constructs.

In my talk, I will largely restrict myself to the literature on the topic of fractions which is already vast. I shall provide brief overviews of what the different strands described have contributed to our understanding of the initial learning of fractions by children in the upper primary and middle grades. I shall also discuss why fractions are difficult and how they are different from whole numbers. My key focus will be on the second research strand described above, which seeks to understand ways of bridging children's informal knowledge analysed through schemes with the symbolic representations that underlie fraction arithmetic. The fraction symbol can be interpreted as an algebraic shorthand for the result of the division operation, which is a powerful integrative idea on the symbolic plane. Exploiting this idea requires that students be familiar with the algebraic aspects of the fraction notation: that numbers and quantities can be represented in terms of their operational composition, and not merely by the canonical forms familiar to them from whole number arithmetic (Subramaniam & Banerjee, in press). This will allow one to make important connections between research on the learning of fractions and the research on the learning of algebra.

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